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The Weakly Interacting Massive Particle (WIMP) is the main candidate for the relic dark matter. A set of exclusion curves currently obtained for cross sections of the spin-dependent WIMP-proton and WIMP-neutron interaction is given. A two-orders-of-magnitude improvement of the sensitivity of the dark matter experiment is needed to reach the SUSY predictions for relic neutralinos. It is noted that near-future experiments with the high-spin isotope ^{73}Ge can yield a new important constraint on the neutralino-neutron effective coupling and the SUSY parameter space.

A. Introduction

Nowadays the main efforts in the direct dark matter search experiments are concentrated in the field of so-called spin-independent (or scalar) interaction of a dark matter particle or the Weakly Interacting Massive Particle (WIMP), with a nucleus. The lightest supersymmetric (SUSY) particle (LSP) neutralino is assumed here as a best WIMP candidate. It is believed that this spin-independent (SI) interaction of dark matter (DM) particles with nuclei makes a dominant contribution to the expected event rate of detection of these particles. The reason is the strong (proportional to the squared mass of the target nucleus) enhancement of SI WIMP-nucleus interaction. The results currently obtained in the field are usually presented in the form of exclusion curves (see for example Fig. 1). For the fixed mass of the WIMP the values of the cross section due to scalar elastic WIMP-nucleon interaction located above these curves are already excluded experimentally. There is also the DAMA closed contour which corresponds to the first claim for evidence for the dark matter signal due to the positive annual modulation effect [5].

In the paper we consider some aspects of the spin-dependent (or axial-vector) interaction of the DM WIMP with nuclei. There are at least three reasons to think that this spin-dependent (SD) interaction could also be very important. First, contrary to the only one constraint for SUSY models available from scalar WIMP-nucleus interaction, the spin WIMP-nucleus interaction supplies us with two such constraints (see for example [6] and formulas below). Second, one can notice [1, 7] that even with a very accurate DM detector (say, with sensitivity 10^{-5} events/day/kg) which is sensitive only to the WIMP-nucleus scalar interaction (with spinless target nuclei) one can, in principle, miss a DM signal. To safely avoid such a situation one should have a spin-sensitive DM detector, i.e. a detector with spin-non-zero target nuclei. Finally, there is a complicated (and theoretically very interesting) nucleus spin structure, which possesses the so-called long q -tail form-factor behavior for heavy targets and heavy WIMP. Therefore, the SD efficiency to detect a DM signal is much higher than the SI efficiency, especially for the heavy target nucleus and WIMP masses [8].

B. Zero Momentum Transfer

A dark matter event is elastic scattering of a relic neutralino χ (or $\tilde{\chi}$) from a target nucleus A producing a nuclear recoil E_R which can be detected by a suitable detector. The differential event rate in respect to the recoil energy is the subject of experimental measurements. The rate depends on the distribution of the relic neutralinos in the solar vicinity $f(v)$ and the cross section of neutralino-nucleus elastic scattering

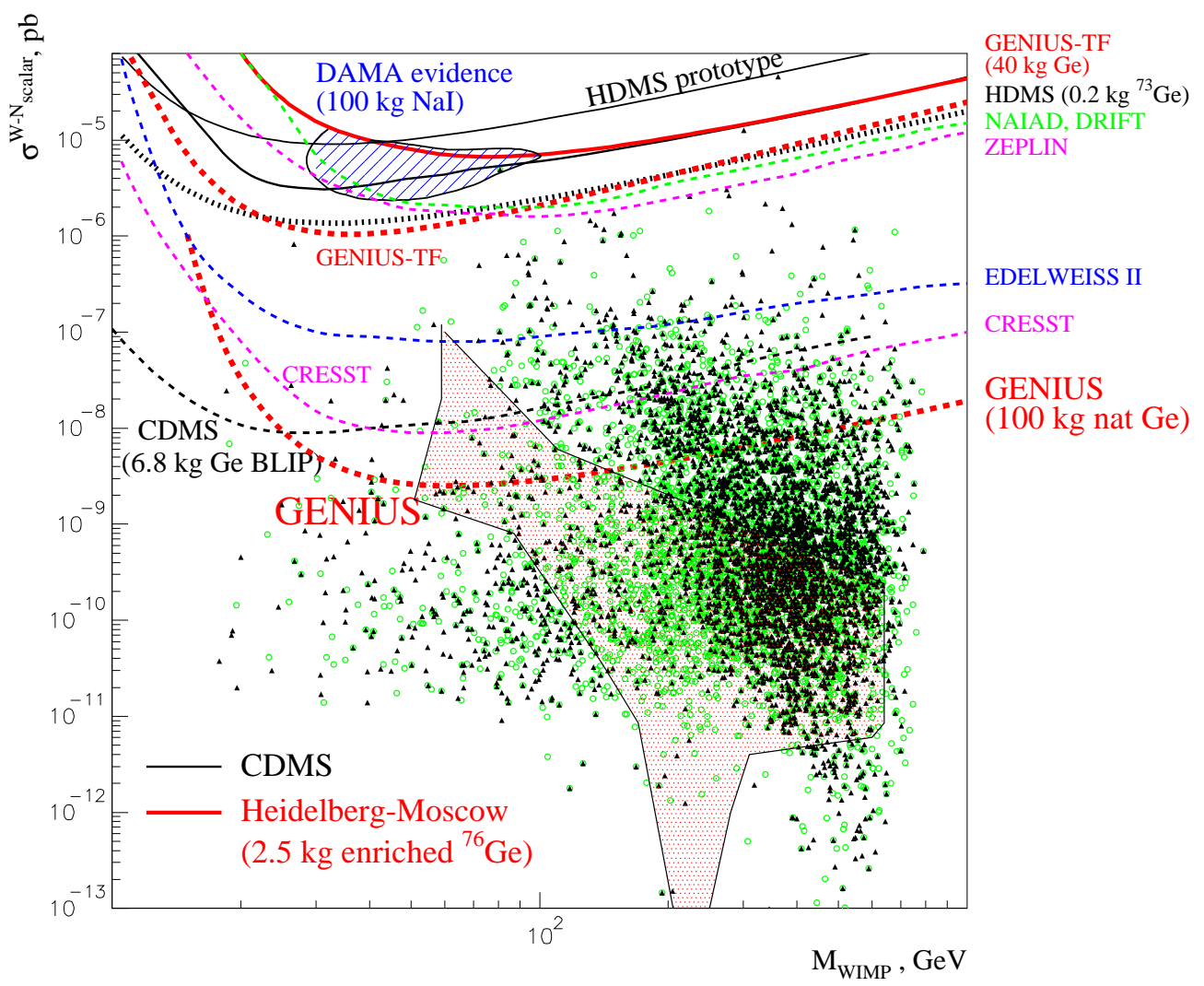


FIG. 1: WIMP-nucleon cross section limits in pb for scalar (spin-independent) interactions as a function of the WIMP mass in GeV. Shown are contour lines of the present experimental limits (solid lines) and of projected experiments (dashed lines). Also shown is the region of evidence published by DAMA. The theoretical expectations are shown by scatter plots (circles and triangles are from [1, 2]) and by grey closed region [3]. From [4].

[9, 10, 11, 12, 13, 14, 15, 16]. The differential event rate per unit mass of the target material has the form

$$\frac{dR}{dE_R} = N \frac{\rho_\chi}{m_\chi} \int_{v_{\min}}^{v_{\max}} dv f(v) v \frac{d\sigma}{dq^2}(v, q^2). \quad (1)$$

The nuclear recoil energy $E_R = q^2/(2M_A)$ is typically about $10^{-6}m_\chi$ and $N = \mathcal{N}/A$ is the number density of target nuclei, where \mathcal{N} is the Avogadro number and A is the atomic mass of the nuclei with mass M_A . The neutralino-nucleus elastic scattering cross section for spin-non-zero ($J \neq 0$) nuclei contains coherent (spin-independent, or SI) and axial (spin-dependent, or SD) terms [8, 17, 18]:

$$\begin{aligned} \frac{d\sigma^A}{dq^2}(v, q^2) &= \frac{\sum |\mathcal{M}|^2}{\pi v^2 (2J+1)} = \frac{S_{\text{SD}}^A(q^2)}{v^2 (2J+1)} + \frac{S_{\text{SI}}^A(q^2)}{v^2 (2J+1)} \\ &= \frac{\sigma_{\text{SD}}^A(0)}{4\mu_A^2 v^2} F_{\text{SD}}^2(q^2) + \frac{\sigma_{\text{SI}}^A(0)}{4\mu_A^2 v^2} F_{\text{SI}}^2(q^2). \end{aligned} \quad (2)$$

The normalized non-zero-momentum-transfer nuclear form-factors

$$F_{\text{SD,SI}}^2(q^2) = \frac{S_{\text{SD,SI}}^A(q^2)}{S_{\text{SD,SI}}^A(0)} \quad (F_{\text{SD,SI}}^2(0) = 1), \quad (3)$$

are defined via nuclear structure functions [8, 17, 18]

$$S_{\text{SI}}^A(q) = \sum_{L \text{ even}} |\langle J | \mathcal{C}_L(q) | J \rangle|^2 \simeq |\langle J | \mathcal{C}_0(q) | J \rangle|^2, \quad (4)$$

$$S_{\text{SD}}^A(q) = \sum_{L \text{ odd}} (|\langle N | \mathcal{T}_L^{el5}(q) | N \rangle|^2 + |\langle N | \mathcal{L}_L^5(q) | N \rangle|^2). \quad (5)$$

The transverse electric $\mathcal{T}^{el5}(q)$ and longitudinal $\mathcal{L}^5(q)$ multipole projections of the axial vector current operator, scalar function $\mathcal{C}_L(q)$ are given in the form

$$\begin{aligned} \mathcal{T}_L^{el5}(q) &= \frac{1}{\sqrt{2L+1}} \sum_i \frac{a_0 + a_1 \tau_3^i}{2} \left[-\sqrt{L} M_{L,L+1}(q\vec{r}_i) + \sqrt{L+1} M_{L,L-1}(q\vec{r}_i) \right], \\ \mathcal{L}_L^5(q) &= \frac{1}{\sqrt{2L+1}} \sum_i \left(\frac{a_0}{2} + \frac{a_1 m_\pi^2 \tau_3^i}{2(q^2 + m_\pi^2)} \right) \left[\sqrt{L+1} M_{L,L+1}(q\vec{r}_i) + \sqrt{L} M_{L,L-1}(q\vec{r}_i) \right], \\ \mathcal{C}_L(q) &= \sum_{i, \text{ nucleons}} c_0 j_L(qr_i) Y_L(\hat{r}_i), \quad \mathcal{C}_0(q) = \sum_i c_0 j_0(qr_i) Y_0(\hat{r}_i), \end{aligned}$$

where $a_{0,1} = a_n \pm a_p$ (see (10)) and $M_{L,L'}(q\vec{r}_i) = j_{L'}(qr_i) [Y_{L'}(\hat{r}_i) \vec{\sigma}_i]^L$ [8, 17, 18]. The nuclear SD and SI cross sections at $q = 0$ (in (2)) have the forms

$$\sigma_{\text{SI}}^A(0) = \frac{4\mu_A^2 S_{\text{SI}}(0)}{(2J+1)} = \frac{\mu_A^2}{\mu_p^2} A^2 \sigma_{\text{SI}}^p(0), \quad (6)$$

$$\sigma_{\text{SD}}^A(0) = \frac{4\mu_A^2 S_{\text{SD}}(0)}{(2J+1)} = \frac{4\mu_A^2}{\pi} \frac{(J+1)}{J} \left\{ a_p \langle \mathbf{S}_p^A \rangle + a_n \langle \mathbf{S}_n^A \rangle \right\}^2 \quad (7)$$

$$= \frac{\mu_A^2}{\mu_{p,n}^2} \frac{(J+1)}{3J} \left\{ \sqrt{\sigma_{\text{SD}}^p(0)} \langle \mathbf{S}_p^A \rangle + \text{sign}(a_p a_n) \sqrt{\sigma_{\text{SD}}^n(0)} \langle \mathbf{S}_n^A \rangle \right\}^2. \quad (8)$$

Here $\mu_A = \frac{m_\chi M_A}{m_\chi + M_A}$ is the reduced neutralino-nucleus mass. The zero-momentum-transfer proton and neutron SI and SD cross sections

$$\sigma_{\text{SI}}^p(0) = 4 \frac{\mu_p^2}{\pi} c_0^2, \quad c_0 \equiv c_0^{(p,n)} = \sum_q \mathcal{C}_q f_q^{(p,n)}; \quad (9)$$

$$\sigma_{\text{SD}}^{p,n}(0) = 12 \frac{\mu_{p,n}^2}{\pi} a_{p,n}^2, \quad a_n = \sum_q \mathcal{A}_q \Delta_q^{(p)}, \quad a_p = \sum_q \mathcal{A}_q \Delta_q^{(n)} \quad (10)$$

depend on the effective neutralino-quark scalar \mathcal{C}_q and axial-vector \mathcal{A}_q couplings from the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \sum_q (\mathcal{A}_q \cdot \bar{\chi} \gamma_\mu \gamma_5 \chi \cdot \bar{q} \gamma^\mu \gamma_5 q + \mathcal{C}_q \cdot \bar{\chi} \chi \cdot \bar{q} q) + \dots \quad (11)$$

and on the spin ($\Delta_q^{(p,n)}$) and mass ($f_q^{(p,n)}$) structure of nucleons. The factors $\Delta_q^{(p,n)}$ parametrize the quark spin content of the nucleon and are defined by the relation $2\Delta_q^{(n,p)} s^\mu \equiv \langle p, s | \bar{\psi}_q \gamma^\mu \gamma_5 \psi_q | p, s \rangle_{(p,n)}$. The total nuclear spin (proton, neutron) operator is defined as follows

$$\mathbf{S}_{p,n} = \sum_i^A \mathbf{s}_{p,n}(i), \quad (12)$$

where i runs over all nucleons. Further the convention is used that all angular momentum operators are evaluated in their z -projection in the maximal M_J state, e.g.

$$\langle \mathbf{S} \rangle \equiv \langle N | \mathbf{S} | N \rangle \equiv \langle J, M_J = J | S_z | J, M_J = J \rangle. \quad (13)$$

Therefore $\langle \mathbf{S}_{p(n)} \rangle$ is the spin of the proton (neutron) averaged over all nucleons in the nucleus A . The cross sections at zero momentum transfer show strong dependence on the nuclear structure of the ground state [19, 20, 21].

The relic neutralinos in the halo of our Galaxy have a mean velocity of $\langle v \rangle \simeq 300 \text{ km/s} = 10^{-3}c$. When the product $q_{\text{max}}R \ll 1$, where R is the nuclear radius and $q_{\text{max}} = 2\mu_A v$ is the maximum momentum transfer in the $\tilde{\chi}A$ scattering, the matrix element for the spin-dependent $\tilde{\chi}A$ scattering reduces to a very simple form (*zero momentum transfer limit*) [20, 21]:

$$\mathcal{M} = C \langle N | a_p \mathbf{S}_p + a_n \mathbf{S}_n | N \rangle \cdot \mathbf{s}_{\tilde{\chi}} = C \Lambda \langle N | \mathbf{J} | N \rangle \cdot \mathbf{s}_{\tilde{\chi}}. \quad (14)$$

Here $\mathbf{s}_{\tilde{\chi}}$ is the spin of the neutralino, and

$$\Lambda = \frac{\langle N | a_p \mathbf{S}_p + a_n \mathbf{S}_n | N \rangle}{\langle N | \mathbf{J} | N \rangle} = \frac{\langle N | (a_p \mathbf{S}_p + a_n \mathbf{S}_n) \cdot \mathbf{J} | N \rangle}{J(J+1)}. \quad (15)$$

It is seen that the χ couples to the spin carried by the protons and the neutrons. The normalization C involves the coupling constants, masses of the exchanged bosons and various LSP mixing parameters that have no effect upon the nuclear matrix element [22]. In the limit of zero momentum transfer $q = 0$ the spin structure function (5) reduces to

$$S_{\text{SD}}^A(0) = \frac{2J+1}{\pi} \Lambda^2 J(J+1). \quad (16)$$

Perhaps the first model to estimate the spin content in the nucleus for the dark matter search was the independent single-particle shell model (**ISPSM**) used originally by Goodman and Witten [23] and later in [11, 24, 25]. The ground state value of the nuclear total spin J can be described by those of one extra nucleon interacting with the effective potential of the nuclear core.

There are nuclear structure calculations (including non-zero-momentum approximation) for spin-dependent neutralino interaction with helium ^3He [26]; fluorine ^{19}F [19, 26, 27]; sodium ^{23}Na [19, 20, 26, 27]; aluminium ^{27}Al [21]; silicon ^{29}Si [17, 19, 27]; chlorine ^{35}Cl [17]; potassium ^{39}K [21]; germanium ^{73}Ge [17, 28]; niobium ^{93}Nb [29]; iodine ^{127}I [20]; xenon ^{129}Xe [20] and ^{131}Xe [8, 20, 30]; tellurium ^{123}Te [30] and ^{125}Te [20]; lead ^{208}Pb [26, 31]. The zero-momentum case is also investigated for Cd, Cs, Ba and La in [30, 32, 33].

There are several approaches (advocated by a few groups of researchers) to the more accurate calculation of the nuclear structure effects relevant to the dark matter detection. To the best of our knowledge an almost full list of the models includes the Odd Group Model (**OGM**) of Engel and Vogel [34] and their extended OGM (**EOGM**) [18, 34]; Interacting Boson Fermion Model (**IBFM**) of Iachello, Krauss, and Maino [33]; Theory of Finite Fermi Systems (**TFFS**) of Nikolaev and Klapdor-Kleingrothaus [35]; Quasi Tamm-Dancoff Approximation (**QTDA**) of Engel [8]; different shell model treatments (**SM**) by Pacheco and Strottman [32]; by Engel, Pittel, Ormand and Vogel [29] and Engel, Ressel, Towner and Ormand, [21], by Ressel et al. [17] and Ressel and Dean [20]; by Kosmas, Vergados et al. [19, 26, 31]; so-called “**hybrid**” model of Dimitrov, Engel and Pittel [28] and perturbation theory (**PT**) based on calculations of Engel et al. [21].

C. Spin constraints

For the spin-zero nuclear target the experimentally measured event rate (1) of direct DM particle detection via formula (2) is connected with zero-momentum WIMP-proton (for the neutron the cross section is the

same) cross section (7). The zero momentum scalar WIMP-proton (neutron) cross section $\sigma_{\text{SI}}^p(0)$ can be expressed through effective neutralino-quark couplings \mathcal{C}_q (11) by means of expression (9). These couplings \mathcal{C}_q (as well as \mathcal{A}_q) can be directly connected with the fundamental parameters of a SUSY model such as $\tan\beta$, $M_{1,2}$, μ , masses of sfermions and Higgs bosons, etc. Therefore experimental limitations on the spin-independent neutralino-nucleon cross section supply us with a constraint on the fundamental parameters of an underlying SUSY model.

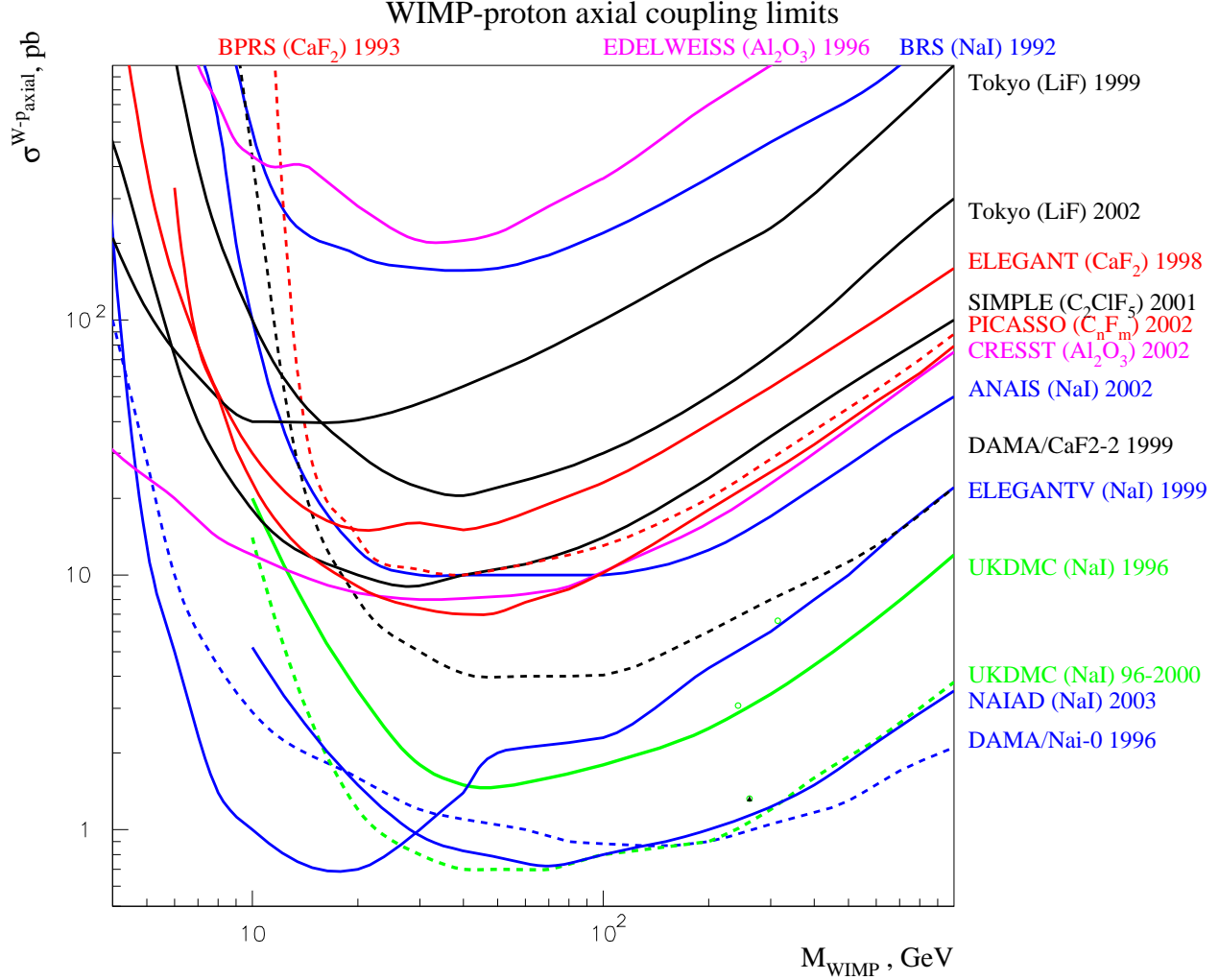


FIG. 2: Full set of currently available exclusion curves for spin-dependent WIMP-proton cross sections (σ_{SD}^p as a function of WIMP mass). The curves are obtained from [36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59].

In the case of the spin-dependent WIMP-nucleus interaction from measured differential rate (1) one first extracts limitation for $\sigma_{\text{SD}}^A(0)$ and therefore has in principle two constraints [6] for the neutralino-proton a_p and neutralino-neutron a_n spin effective couplings as follows from relation (7). From (7) one can also see that contrary to spin-independent case (6) there is no factorization of the nuclear structure for $\sigma_{\text{SD}}^A(0)$. Both proton $\langle \mathbf{S}_p^A \rangle$ and neutron $\langle \mathbf{S}_n^A \rangle$ spin contributions simultaneously entering formula (7) for the SD WIMP-nuclear cross section $\sigma_{\text{SD}}^A(0)$.

In the earlier considerations based on the OGM [18, 34] one assumed that the nuclear spin is carried by

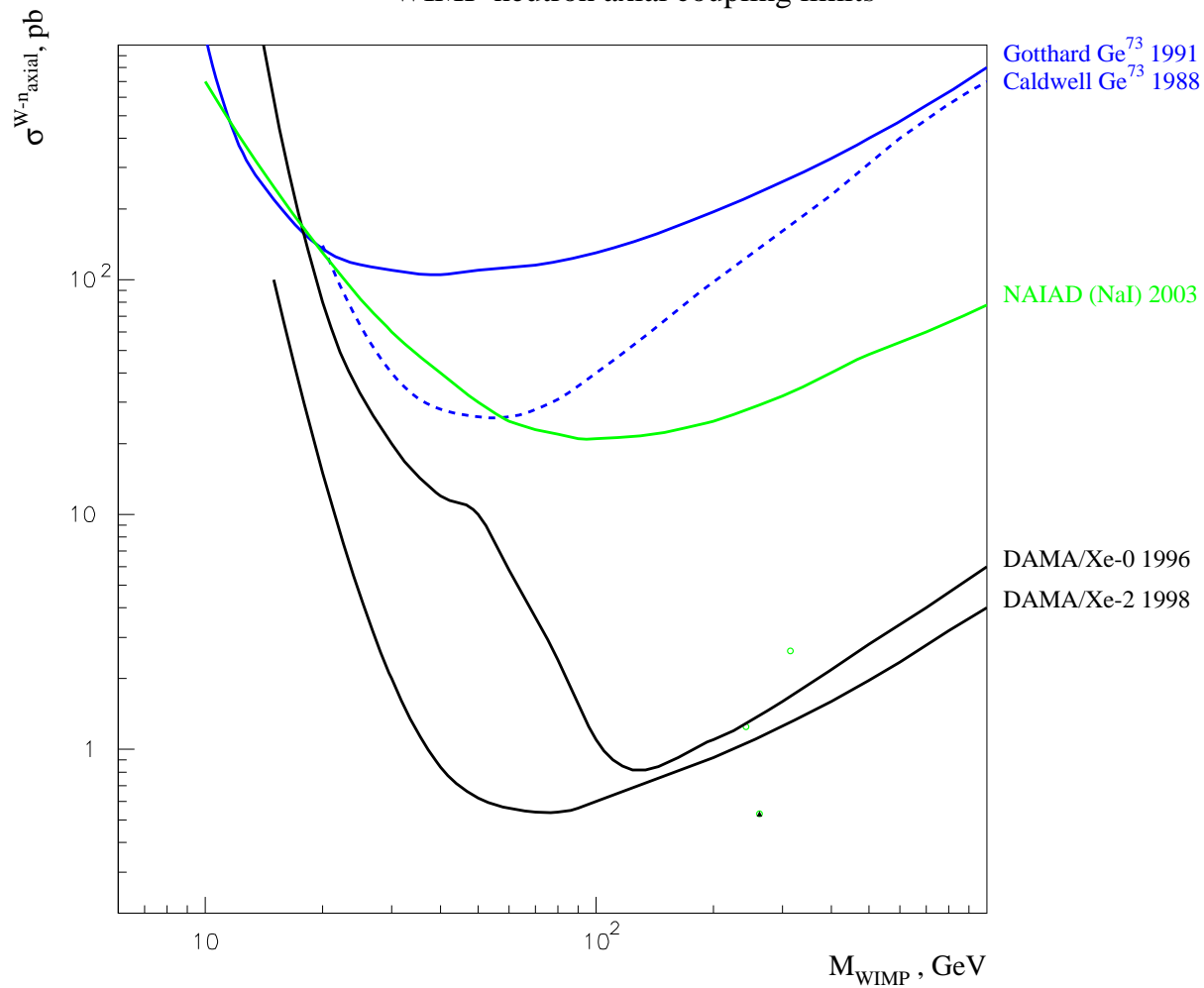


FIG. 3: Full set of currently available exclusion curves for spin-dependent WIMP-neutron cross sections (σ_{SD}^n versus WIMP mass). The curves are obtained from [59, 60, 61, 62, 63, 64]. Note that the NAIAD curve corresponds to the sub-dominant contribution, extracted from the p-odd nucleus Na (compare with the relevant NAIAD curve in Fig. 2)

the “odd” unpaired group of protons or neutrons and only one of either $\langle \mathbf{S}_n^A \rangle$ or $\langle \mathbf{S}_p^A \rangle$ is non-zero (the same is true in the ISPSM [11, 23, 24, 25]). In this case all possible target nuclei can naturally be classified into n-odd and p-odd groups. The current experimental situation for the spin-dependent WIMP-**proton** cross section is given in Fig. 2. The data are taken from experiments BRS, (NaI, 1992) [36, 37], BPRS (CaF₂, 1993) [38], EDELWEISS (sapphire, 1996) [39], DAMA (NaI, 1996) [40], DAMA (CaF₂, 1999) [41, 42], UKDMS (NaI, 1996) [43, 44, 45, 46], ELEGANT (CaF₂, 1998) [47], ELEGANT (NaI, 1999) [48, 49], Tokio (LiF, 1999, 2002) [50, 51, 52, 53, 54], SIMPLE (C₂ClF₅, 2001) [55], CRESST (Al₂O₃, 2002) [56], PICASSO (C_nF_m, 2002) [57], ANAIS (NaI, 2002) [58] and NAIAD (NaI, 2003) [59]. The current experimental situation for the spin-dependent WIMP-**neutron** cross section is given in Fig. 3. The data are taken from the first experiments with natural Ge (1988, 1991) [60, 61], xenon (DAMA/Xe-0,2) [62, 63, 64] and sodium iodide (NAIAD) [59]. In the future one can also expect exclusion curves for the SD cross section, for example, from the EDELWEISS and CDMS experiments with natural germanium bolometric detectors.

From Fig. 4 one can conclude that an about two-orders-of-magnitude improvement of the current DM

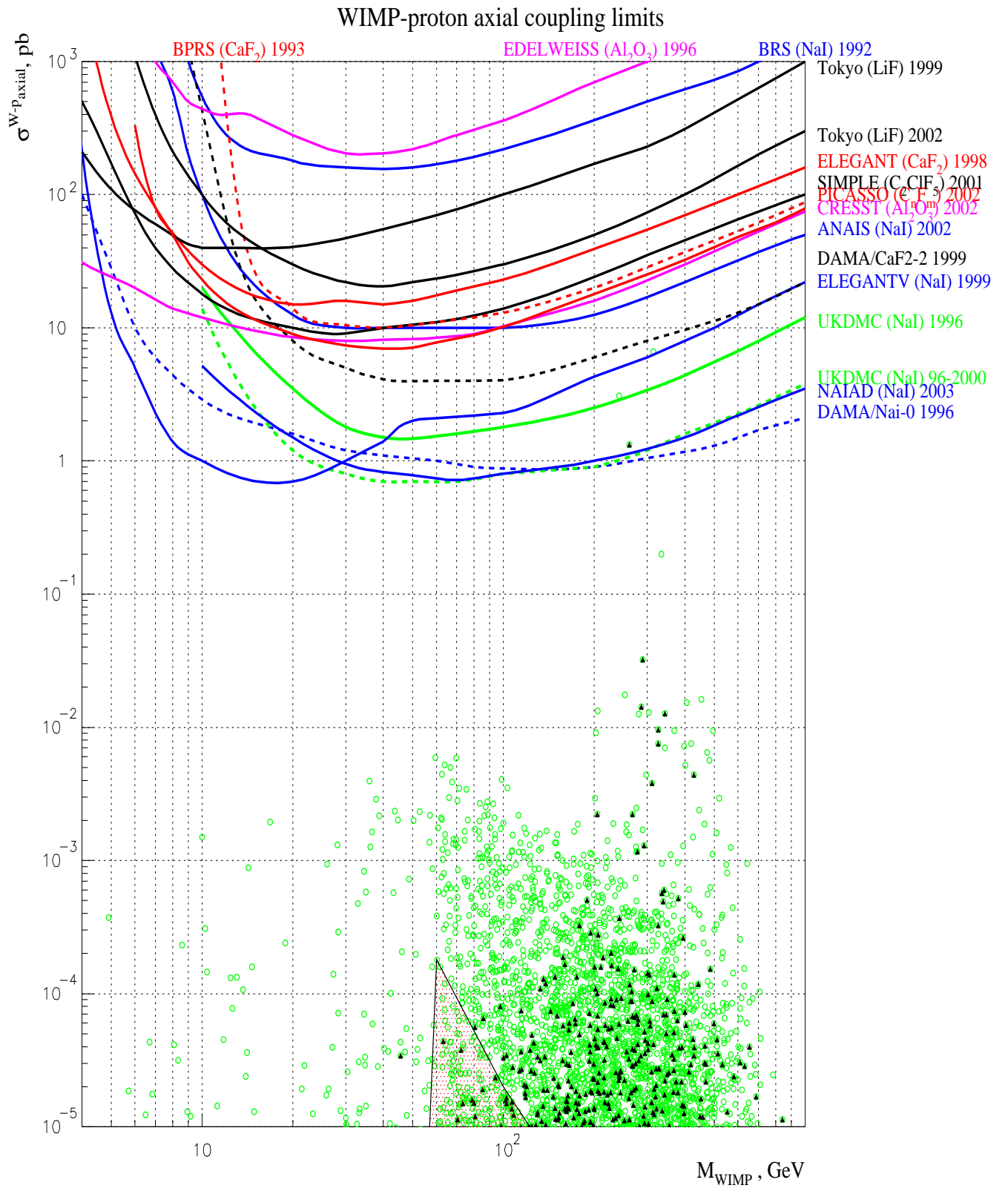


FIG. 4: The same as in Fig. 2, but with the theoretical scatter plot from [2], obtained in the effMSSM with all coannihilation channels included (green circles) and with $0.1 < \Omega h^2 < 0.3$ (black triangles). The triangle-like shaded area is taken from [3].

experiment sensitivities is needed to reach the SUSY predictions for the σ_{SD}^p provided the SUSY lightest neutralino (LSP) is the best WIMP particle candidate. There is the same situation for the σ_{SD}^n .

Further more accurate calculations of [8, 17, 19, 20, 21, 26, 28, 29, 31, 32, 33] demonstrate that contrary to the simplified odd-group approach both $\langle \mathbf{S}_p^A \rangle$ and $\langle \mathbf{S}_n^A \rangle$ differ from zero, but nevertheless one of these spin quantities always dominates ($\langle \mathbf{S}_p^A \rangle \ll \langle \mathbf{S}_n^A \rangle$, or $\langle \mathbf{S}_n^A \rangle \ll \langle \mathbf{S}_p^A \rangle$). If together with the dominance like $\langle \mathbf{S}_{p(n)}^A \rangle \ll \langle \mathbf{S}_{n(p)}^A \rangle$ one would have the WIMP-proton and WIMP-neutron couplings of the same order of magnitude (not $a_{n(p)} \ll a_{p(n)}$), the situation could look like that in the odd-group model. Nevertheless it

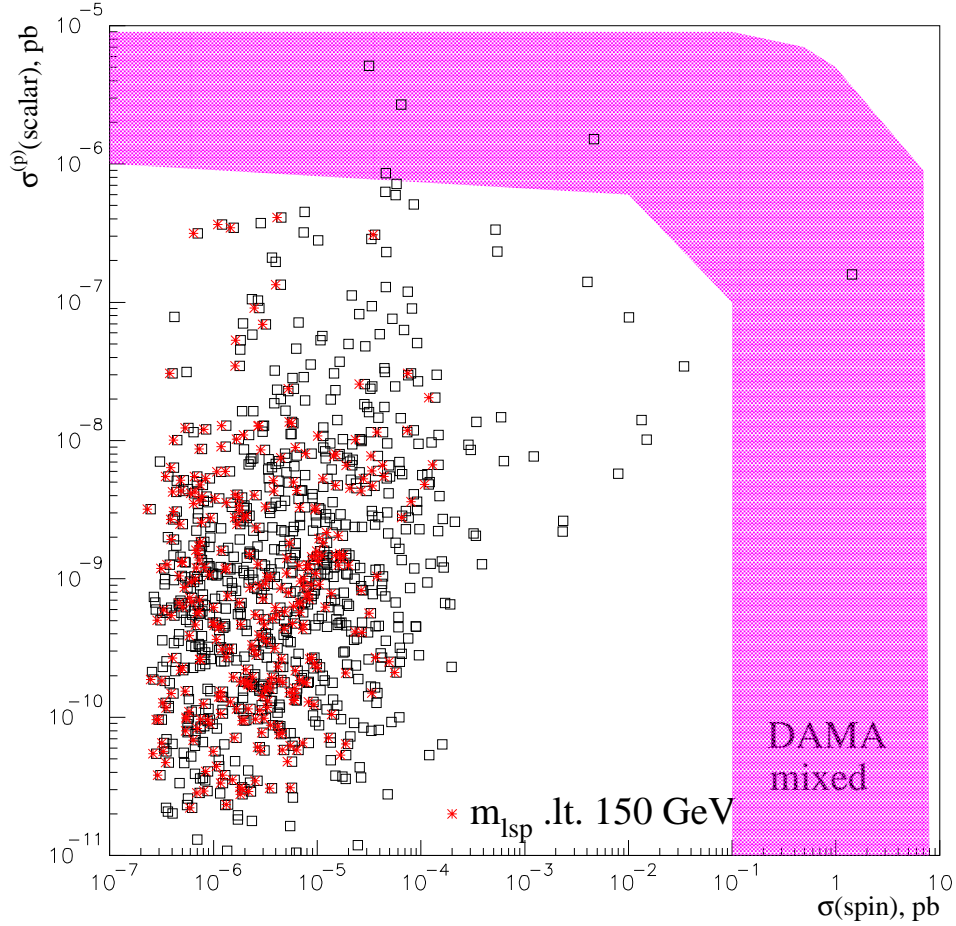


FIG. 5: The DAMA/NaI region from the WIMP annual modulation signature (57986 kg day) in the $(\xi\sigma_{\text{SI}}, \xi\sigma_{\text{SD}})$ space for $40 \text{ GeV} < m_{\text{WIMP}} < 110 \text{ GeV}$ covers all four particular couplings ($\theta = 0, \theta = \pi/4, \theta = \pi/2$ and $\theta = 2.435$ rad) reported in [66]. Scatter plots give correlations between $\sigma_{\text{SI}}^p(0)$ and σ_{SD} in effMSSM ($\xi = 1$ is assumed) with the requirement for neutralino relic density $0.1 < \Omega h^2 < 0.3$ and all coannihilation channels included [2]. Red stars correspond to an assumption that $m_{\text{LSP}} < 150 \text{ GeV}$.

was shown in [65] that in the general SUSY model one can meet a case when $a_{n(p)} \ll a_{p(n)}$. To solve the problem (to separate SD proton and neutron constraints) at least two new approaches were proposed. As the authors of [65] claimed, their method has the advantage that the limits on individual WIMP-proton and WIMP-neutron SD cross sections for a given WIMP mass can be combined to give a model-independent limit on the properties of WIMP scattering from both protons and neutrons in the target nucleus. The method relies on the assumption that the WIMP-nuclear SD cross section can be presented in the form $\sigma_{\text{SD}}^A(0) = \left(\sqrt{\sigma_{\text{SD}|A}^p} \pm \sqrt{\sigma_{\text{SD}|A}^n} \right)^2$, where $\sigma_{\text{SD}|A}^p$ and $\sigma_{\text{SD}|A}^n$ are auxiliary quantities, not directly connected

with measurements. Furthermore, to extract, for example, a constraint on the sub-dominant WIMP-proton spin contribution one should assume the proton contribution dominance for a nucleus whose spin is almost completely determined by the neutron-odd group. It may seem useless, especially because these sub-dominant constraints are always much weaker than the relevant constraints obtained directly with a proton-odd group target.

Another approach of [66] is based on introduction of another auxiliary quantity $\sigma_{\text{SD}} = 12 \frac{\mu_p^2}{\pi} (a_p^2 + a_n^2)$, where $\tan \theta = a_n/a_p$. With these definitions the SD WIMP-proton and WIMP-neutron cross section can be given in the form $\sigma_{\text{SD}}^{n(p)}(0) = \sigma_{\text{SD}} \sin^2 \theta (\cos^2 \theta)$. In Fig. 5 the WIMP-nucleon spin-mixed and scalar couplings allowed by annual modulation signature from the 100-kg DAMA/NaI experiment (57986 kg day) are given by filled region. The region (at 3σ C.L.) in the $(\xi\sigma_{\text{SI}}, \xi\sigma_{\text{SD}})$ space for $40 \text{ GeV} < m_{\text{WIMP}} < 110 \text{ GeV}$ covers all four particular couplings ($\theta = 0$, $\theta = \pi/4$, $\theta = \pi/2$ and $\theta = 2.435 \text{ rad}$) reported in [66]. Scatter plots give $\sigma_{\text{SI}}^p(0)$ versus σ_{SD} in effMSSM with $0.1 < \Omega h^2 < 0.3$ and all coannihilation channels included from [2] under the assumption of $\xi = 1$. Red stars correspond to the same calculations but with $m_{\text{LSP}} < 150 \text{ GeV}$. In this mixed case the limits for the spin couplings depend on assumptions about the scalar coupling, and the relevant exclusion curve for the spin-dependent WIMP-proton cross section (not given in Fig. 2) can not be simply extracted from these mixed results of [67].

D. The role of the germanium-73 isotope

Comparing the number of exclusion curves in Fig. 2 and Fig. 3 one can easily see that there are many measurements with p-odd nuclei and there is a lack of data for n-odd nuclei, i.e. for σ_{SD}^n . Therefore measurements with n-odd nuclei are needed. From our point of view this lack of σ_{SD}^n measurements can be filled with new data expected from the HDMS experiment with the high-spin isotope ^{73}Ge [68]. This isotope looks with a good accuracy like an almost pure n-odd group nucleus with $\langle \mathbf{S}_n \rangle \gg \langle \mathbf{S}_p \rangle$ (Table I). The variation of the $\langle \mathbf{S}_p \rangle$ and $\langle \mathbf{S}_n \rangle$ in the table reflects the level of inaccuracy and complexity of the current nuclear structure calculations.

TABLE I: Zero-momentum spin structure (and predicted magnetic moments μ) of the ^{73}Ge nucleus in different nuclear models. The experimental value of the magnetic moment given in the brackets is used as input in the calculations.

$^{73}\text{Ge} (L_J = G_{9/2})$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	μ (in μ_N)
ISPSM, Ellis-Flores [24, 69]	0	0.5	-1.913
OGM, Engel-Vogel [34]	0	0.23	$(-0.879)_{\text{exp}}$
IBFM, Iachello et al. [33] and [17]	-0.009	0.469	-1.785
IBFM (quenched), Iachello et al. [33] and [17]	-0.005	0.245	$(-0.879)_{\text{exp}}$
TFFS, Nikolaev-Klapdor-Kleingrothaus, [35]	0	0.34	—
SM (small), Ressel et al. [17]	0.005	0.496	-1.468
SM (large), Ressel et al. [17]	0.011	0.468	-1.239
SM (large, quenched), Ressel et al. [17]	0.009	0.372	$(-0.879)_{\text{exp}}$
“Hybrid” SM, Dimitrov et al. [28]	0.030	0.378	-0.920

On the other hand, Fig. 6 shows that for the ratio of a_n to a_p one can have the bounds

$$0.55 < \left| \frac{a_n}{a_p} \right| < 0.8.$$

The scatter plots in Fig. 6 were obtained in effMSSM [2] when all coannihilation channels were included. The blue squares (black points) were calculated with (without) the relic neutralino density constraint $0.1 < \Omega h^2 < 0.3$. Therefore in the model the couplings are almost the same and one can safely neglect the $\langle \mathbf{S}_p^A \rangle$ -spin contribution in the analysis of the DM data with the ^{73}Ge target (for which $\langle \mathbf{S}_p^A \rangle \ll \langle \mathbf{S}_n^A \rangle$).

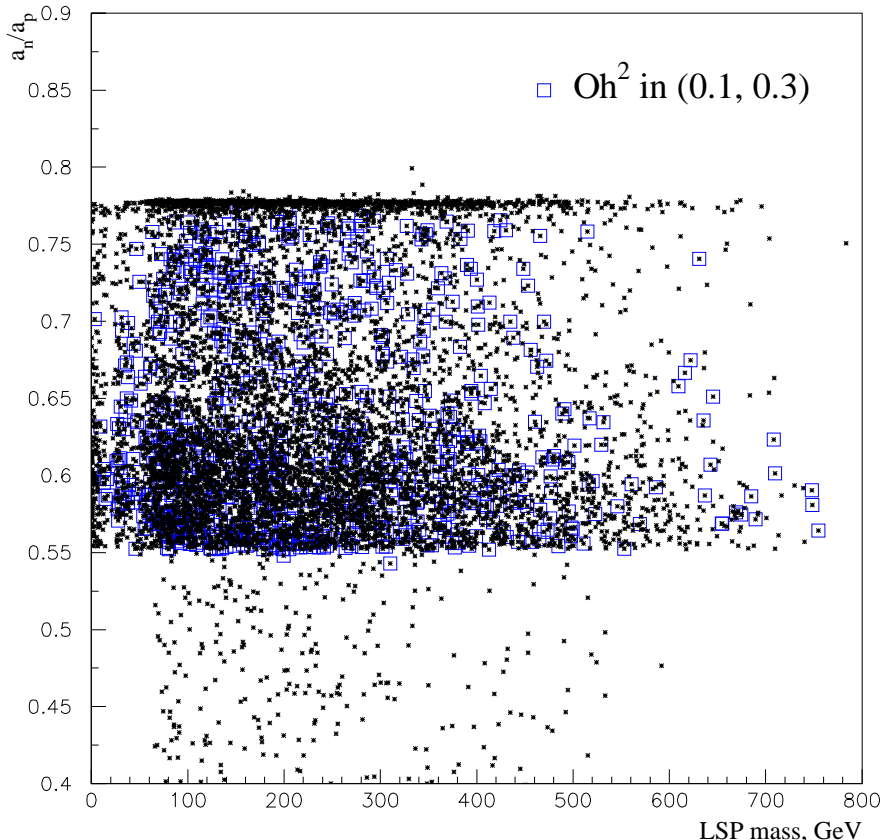


FIG. 6: The scatter plot (dots) gives the ratio of the neutralino-neutron a_n and neutralino-proton a_p spin couplings in the effMSSM [2]. Boxes correspond to the relic neutralino density constraint $0.1 < \Omega h^2 < 0.3$ in the same model.

We would like to advocate the old odd-group-like approach for experiments with germanium detectors. Of course, from measurements with ^{73}Ge one can extract not only the dominant constraint for WIMP-nucleon coupling a_n (or σ_{SD}^n) but also the constraint for the sub-dominant WIMP-proton coupling a_p (or σ_{SD}^p) using the approach of [65]. Nevertheless, the latter constraint will be much weaker in comparison with the constraints from p-odd group nuclear targets, like ^{19}F or NaI . The fact illustrates the NAIAD (NaI , 2003) curve in Fig. 3, which corresponds to the sub-dominant WIMP-neutron spin contribution extracted from the p-odd nucleus Na .

E. Finite Momentum Transfer

As $m_{\tilde{\chi}}$ becomes larger, the product qR ceases to be negligible and the finite momentum transfer limit must be considered for heavier nuclei. With the isoscalar coupling constant $a_0 = a_n + a_p$ and the corresponding isovector coupling constant $a_1 = a_p - a_n$ one splits $S_{\text{SD}}^A(q)$ into a pure isoscalar, S_{00} , a pure isovector, S_{11} , and an interference term, S_{01} [17, 20]:

$$S_{\text{SD}}^A(q) = a_0^2 S_{00}^A(q) + a_1^2 S_{11}^A(q) + a_0 a_1 S_{01}^A(q). \quad (17)$$

The differential SD event rate has the form

$$\frac{dR_{\text{SD}}^A}{dq^2} = \frac{\rho}{m_{\tilde{\chi}} m_A} \int v dv f(v) \frac{8G_F^2}{(2J+1)v^2} S_{\text{SD}}^A(q). \quad (18)$$

Comparing the differential rate (18) together with the spin structure functions of (17) with the observed recoil spectra for different targets (Ge, Xe, F, NaI, etc) one can directly and simultaneously restrict both isoscalar and isovector neutralino-nucleon effective couplings $a_{0,1}$. These constraints will impose most model-independent restrictions on the MSSM parameter space free from any assumption of [65, 66]. Perhaps, it would be the best to fit all data directly [65] in terms of neutralino proton and neutron effective spin couplings $a_{0,1}$ or $a_{p,n}$ (see for example analysis of [54]) and not to use such spin quantities as $\sigma_{\text{SD}}^{p,n}$ and σ_{SD} .

Another attractive feature of the spin-dependent WIMP-nucleus interaction is the q -dependence of SD structure function (17). One knows that the ratio of SD to SI rate in the ^{73}Ge detector grows with the WIMP mass [1, 7]. The growth is much greater for heavy target isotopes like xenon. The reason is the different behavior of the spin and scalar structure functions with increasing momentum transfer. For example, the xenon SI structure function vanishes for $q^2 \approx 0.02 \text{ GeV}^2$, but the SD structure function is a non-zero constant in the region (Fig. 7).

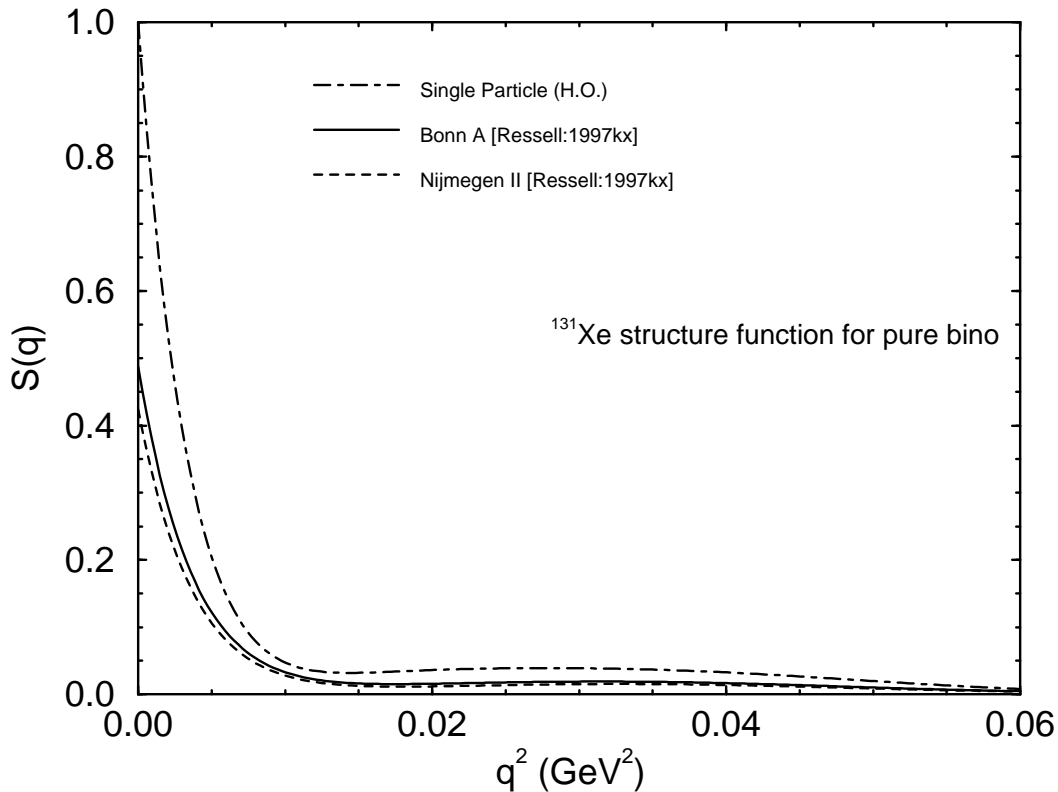


FIG. 7: The ^{131}Xe structure function for a pure bino. The single-particle structure function has been normalized to $S(0) = 1$. From [20].

As noted by Engel in [8], the relatively long tail of the SD structure function is caused by nucleons near the Fermi surface, which do the bulk of the scattering. The core nucleons, which dominate the SI nuclear coupling, contribute much less at large q . Therefore the SD efficiency for detection of a DM signal is higher than the SI efficiency, especially for very heavy neutralinos.

The idea of this review paper is to attract attention to the role of the spin-dependent WIMP-nucleus interaction in the dark matter search experiments. The importance of this interaction is discussed. The fullest possible set of currently available exclusion curves for spin-dependent WIMP-proton and WIMP-neutron cross sections is given in Fig. 2 and Fig. 3. Nowadays about two-orders-of-magnitude improvement of the current DM experiment sensitivities is needed to reach the SUSY predictions for the $\sigma_{\text{SD}}^{p,n}$. It is noted that a near-future experiment like HDMS [68] with the high-spin isotope ^{73}Ge being an almost pure n-odd nucleus can fill in this gap and can supply us with new important constraints for SUSY models.

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